Closed Loop Control of DC Drives

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Contents

- DC Motor Model
- Current Loop Design
- Speed Loop Design
- Digital Implementation Essentials



$$L_a \frac{di_a}{dt} + R_a \cdot i_a + e_b = v_a$$
$$e_b = c_1 \cdot \phi \cdot \omega$$

$$J\frac{d\omega}{dt} = m_d - m_l$$
$$m_d = c_2 \cdot \phi \cdot i_a$$
$$e_b \cdot i_a = m_d \cdot \omega$$

$$c_1 \cdot \phi \cdot \omega \cdot i_a = c_2 \cdot \phi \cdot i_a \cdot \omega$$

Where, $c_1 = c_2 = c$ for proper SI units.

$$\therefore e_b = c \cdot \phi \cdot \omega, \quad \& \quad m_d = c \cdot \phi \cdot i_a$$

For separately excited dc motor, ϕ is constant. Hence, $c \cdot \phi = c_{\phi}$ Where, c_{ϕ} is the Back-emf constant (V/(rad/s)), as well as the Torque constant (N-m/A).



 \mathcal{K}_a -> Armature Resistance e_b -> Back-emf J-> Moment of Inertia $m_d \& m_l$ ->Developed and Load Torques **Dynamic Equations**

 L_{c}

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- Time-domain block-diagram representation
- s-domain block-diagram representation
- One mechanical Output (ω) and One electrical Output (i_a)
- One mechanical Input (m_l) and One electrical input (v_a)

Transfer Functions

•
$$F_1(s) = \frac{\Omega(s)}{V_a(s)}$$

•
$$F_2(s) = \frac{I_a(s)}{V_a(s)}$$

•
$$F_3(s) = \frac{\Omega(s)}{M_l(s)}$$

•
$$F_4(s) = \frac{I_a(s)}{M_l(s)}$$

$$\begin{pmatrix} \frac{JR_a}{c_{\phi}^2} \end{pmatrix} \cdot s \cdot (1 + sT_a) + 1$$

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From these transfer functions, the various time-responses as well as steady-state responses can be obtained.

(Final Value Theorem)

$$F_{1}(s) = \frac{1/c_{\phi}}{\left(\frac{JR_{a}}{c_{\phi}^{2}}\right) \cdot s \cdot (1+sT_{a})+1}$$

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The plant dynamics



The plant dynamics

The Characteristic Equation:

$$s^2 + s\left(\frac{1}{T_a}\right) + \frac{1}{T_a T_{em}}$$

Standard Second-Order Equation:

$$s^2 + 2\zeta\omega_n s + {\omega_n}^2$$

In terms of motor-parameters:

$$\omega_n = \frac{c_{\phi}}{\sqrt{JL_a}}$$
$$\zeta = \frac{1}{2} \frac{R_a}{c_{\phi}} \sqrt{\frac{J}{L_a}}$$

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Standard Second-Order Equation:

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The roots of the characteristic equation:

$$s_{1,2} = -\frac{1}{2T_a} \pm \frac{1}{2T_a} \sqrt{1 - \frac{4T_a}{T_{em}}}$$

For $\frac{T_{em}}{T_a} < 4$, the plant (motor) is under-damped. The motor parameters R_a , L_a , J etc can be obtained from experiments.

Why Cascade Control?

۲	Requirements of a drive									
	Position control									
	• Speed control									
	Acceleration control									
	• Torque/current control									
	Overload protection									
	• Four quadrant operation									
٠	Cascade control									
	• Torque -> Acceleration -> Speed -> Position									
	• Bandwidth requirements: High \rightarrow Low									
	• Design steps: First -> Last									
	• Slow response: Use feed-forward in references!									

Closed-loop Block diagram



- Two loops
- Design the faster inner-loop first : The current loop
- The relevant transfer function: $F_2(s) = \frac{I_a(s)}{V_a(s)}$

Current-loop Block diagram



- Zero steady state error
- Better stability prospects compared to I controller



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P-I Implementation



P-I Implementation



Converter Gain: Full Bridge



The switching law:

 S_4 ON for $V_{tri} > V_{ref}$

 S_1 ON for $V_{tri} < V_{ref}$

 S_2 ON for $V_{tri} > -V_{ref}$

 S_3 ON for $V_{tri} > -V_{ref}$

From the geometry, we see that:

$$V_a = \frac{V_{dc}}{V_p} \cdot V_{ref}$$

$$K_A = \frac{V_{DC}}{V_p}$$

The average delay, $T_d = \frac{T_c}{2}$

Current Sensor Transfer Function



Current Sensor Transfer Function



Current Loop



Current Loop



Current-loop



Current-loop

With K_c chosen as above, the current-loop T.F becomes:

$$\frac{I_a(s)}{I_a^*(s)} \approx \frac{1}{k_2} \cdot \frac{(1+sT_2)}{2\sigma^2 s^2 + 2\sigma s + 1}$$

The zero $(1 + sT_2)$ causes undesirable overshoot, and can be corrected by cancelling with a pole.

The current loop Band-width is:

$$BW\approx \frac{1}{2\sigma}$$

Speed-loop



$$\frac{I_a(s)}{I_a^*(s)} \approx \frac{1}{k_2} \cdot \frac{(1+sT_2)}{2\sigma^2 s^2 + 2\sigma s + 1} \approx \frac{1}{k_2} \frac{1}{1+2\sigma s}$$

Speed-loop





Symmetric Optimum method is used here.

$$G(s)H(s) = K_{\omega} \frac{1 + sT_{\omega}}{sT_{\omega}} \cdot \frac{1/k_2}{1 + 2\sigma s} \cdot \frac{c_{\phi}}{sJ} \cdot \frac{k_1}{1 + sT_1}$$
$$G(s)H(s) \approx K_{\omega} \frac{1 + sT_{\omega}}{sT_{\omega}} \cdot \frac{c_{\phi}}{k_2} \cdot \frac{1}{sJ} \cdot \frac{k_1}{1 + \delta s}$$

where, $\delta = 2\sigma + T_1$.





 $\omega_z = \frac{1}{T_{\rm er}}$ Choose the speed-contoller time-constant as: $T_{\omega} = a^2 \delta$, where a > 1Choose the cross-over frequency as the harmonic mean of the pole and zero frequencies. $\omega_c = \sqrt{\omega_z \omega_p}$ $|GH(j\omega)| = k_{\omega} \frac{\sqrt{1 + (\omega T_{\omega})^2}}{\omega T_{\omega}} \cdot \frac{1/k_2}{\sqrt{1 + (\omega \delta)^2}} \cdot \frac{c_{\phi}}{J_{\omega}} \cdot K_1$ at $\omega = \omega_c$, $|GH(j\omega)| = 1$ (Gain-cross over frequency). $|GH(j\omega)| = 1 = \frac{K_{\omega} \cdot c_{\phi} \cdot K_1}{k_{\alpha} I} \cdot a\delta$ Or. Which gives us, Or, $K_{\omega} = \frac{k_2 J}{c_1 + K_1 q \delta}$ a=2 is an optimum value. Control of DC Drives Dinesh Gopinath (CET) October 4, 2019

The phase margin can be obtained as,

Dr,
$$\phi_M = \arctan\left(a\right) - \arctan\left(\frac{1}{a}\right)$$

Other considerations:

- Feed-forward of back emf
- Rate-of-change limiter for references

Digital Implementation: Fixed-point Vs Floating-Point

- Per-unit systems is adopted for fixed-point implementations
- A base value is chosen for all variables (voltage, frequency, phase, current etc)
- All variables are expressed in p.u
- p.u values are converted in to digital equivalents using fixed-point arithmetic
- Base conversion is done as per accuracy requirements

Digital Implementation: An example

- Let the input voltage be varying in the range $230V \pm 10\%$.
- The maximum voltage is then 357.742V
- If the base voltage is chosen as 360V, the maximum possible voltage is 0.9937 p.u

Digital Implementation: An example

In digital implementation, suppose we use a 12 bit ADC with an input voltage range $\pm 10V$.

The ADC will give digital equivalent values as follows:

ADC	12 bit	p.u for 5V base	p.u for 10V base				
input	Digital	5V base	10 V base				
voltage	output						
+10 V	7FFh	2 p.u	1 p.u				
+5 V	3FFh	1 p.u	0.5 p.u				
+2.5 V	1FFh	0.5 p.u	0.25 p.u				
+1.25 V	FFh	0.25 p.u	0.125 p.u				
0 V	000h	0 p.u	0 p.u				
-1.25 V	F01h	-0.25 p. u	-0.125 p.u				
-2.5 V	E01h	-0.5 p.u	-0.25 p. u				
-5.0 V	C01h	-1.0 p.u	-0.5 p.u				
-10 V	801h	-2.0 p.u	-1.0 p.u				

Digital Implementation: An Example

We may choose the input voltage-sensor in different ways. For example:

- The nominal input voltage range of the ADC is $\pm 10V$ for the maximum expected input voltage
- Solution Constrained ADC East the enough room for the input voltage, and choose $\pm 5V$ as the nominal ADC input voltage range.

In the first case, we must choose the sensor-gain such that the ADC always get a voltage inside its range of $\pm 10V$.

In the second case, we must choose the sensor-gain in such a way that the ADC gets a nominal voltage of 5V when the input voltage is in the nominal range.

Here, however, we can have an unexpected higher voltage signal at the ADC input upto 2 p.u.

Normally, 2 p.u range is kept for signals like current, speed etc.

Some considerations

- Multiplication accuracy
 - Intermediate calculations should be done at higher base values
- Changing the base values
- Sampling frequency

FPGA based digital platform



FPGA platform: features

- 80 digital I/Os
- 16 Analog input channels (with 6.4 μ s A/D conversion time per channel)
- 8 Analog output channels (with a DAC settling time of 80 ns)
- ALTERA EP2C70F672C8 FPGA with 68,416 logic elements
- USB and CAN transceiver interfaces
- On board SRAM (64K×18)
- Three clocks at 20 MHz each
- JTAG interface

FPGA platform: Architecture



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- Werner Leonhard, ?Control of Electrical Drives,? Springer International, Third Ed, 2001

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